Why normal Fermi systems with sufficiently singular interactions do not have a sharp Fermi surface

1995 J. Phys. A: Math. Gen. 28 L571

(http://iopscience.iop.org/0305-4470/28/22/002)

J. Phys. A: Math. Gen. 28 (1995) L571–L576. Printed in the UK

LETTER TO THE EDITOR

Why normal Fermi systems with sufficiently singular interactions do not have a sharp Fermi surface

Peter Kopietz

Institut für Theoretische Physik der Universität Göttingen, Bunsenstraße 9, D-37073 Göttingen, Germany

Received 10 August 1995

Abstract. We use a bosonization approach to show that the momentum distribution n_k of *normal Fermi systems with sufficiently singular interactions is analytic in the vicinity of the* non-interacting Fermi surface. These include singular density–density interactions that diverge in d dimensions stronger than $|q|^{-2(d-1)}$ for vanishing momentum transfer q, but also fermions that are coupled to transverse gauge fields in $d < 3$.

As first noticed by Bares and Wen [1], singular density–density interactions with Fourier transform $f_q \propto |q|^{-\eta}$ destroy in d dimensions the Fermi liquid state for $\eta \geqslant 2(d-1)$. The case $\eta = 2(d-1)$ is marginal and corresponds to a Luttinger liquid, while for $\eta > 2(d-1)$ one obtains normal metals which are neither Fermi liquids nor Luttinger liquids. We shall call these metals *strongly correlated quantum liquids*. The properties of these systems are not very well understood. Certainly strongly correlated quantum liquids cannot be studied by means of conventional many-body perturbation theory, because the perturbative calculation of the self-energy leads to power-law divergences. In the present work we shall use higher-dimensional bosonization to calculate the momentum distribution n_k in these systems. We find that for $\eta > 2(d-1)$ the momentum distribution n_k does not have any singularities, so that a sharp Fermi surface cannot be defined. We then argue that below three dimensions the momentum distribution of electrons that are coupled to transverse gauge fields has also this property.

Bosonization in arbitrary dimensions has recently been discussed by a number of authors [2–8]. The fundamental geometric construction is the subdivision of the Fermi surface into patches of area Λ^{d-1}. With each patch one then associates a 'squat box' K^{α} [4] of radial height $\lambda \ll k_{\rm F}$ and volume $\lambda\Lambda^{d-1}$, and partitions the degrees of freedom close to the Fermi surface into the boxes K^{α}. Here $k_{\rm F}$ is the Fermi wavevector, and α labels the boxes in some convenient ordering. If the size of the patches is chosen small enough, then the curvature of the Fermi surface can be *locally* neglected. The essential motivation for this construction is that it opens the way for a *local linearization* of the non-interacting energy dispersion $\epsilon_{\boldsymbol{k}}$: If $\boldsymbol{k}^{\alpha}$ points to the suitably defined centre of box K^{α}, then for $\boldsymbol{k} \in K^{\alpha}$ we may shift $\boldsymbol{k} = \boldsymbol{k}^{\alpha} + \boldsymbol{q}$, and expand $\epsilon_{\boldsymbol{k}^{\alpha}+\boldsymbol{q}} - \mu \approx \boldsymbol{v}^{\alpha} \cdot \boldsymbol{q}$, where μ is the chemical potential and $\boldsymbol{v}^{\alpha}$ is the local Fermi velocity. At high densities and for interactions that are dominated by small momentum transfers the linearization implies *in arbitrary dimension a large-scale cancellation between self-energy and vertex corrections* (generalized closed loop theorem [8]), so that the entire perturbation series can be summed in a controlled way. The final result for the Matsubara–Green function $G(\boldsymbol{k}, {\rm i}\tilde{\omega}_n)$ of the interacting many-body system is then

$$G(\boldsymbol{k}, \mathrm{i}\tilde{\omega}_n) = \sum_\alpha \Theta^\alpha(\boldsymbol{k}) \int \mathrm{d}\boldsymbol{r} \int_0^\beta \mathrm{d}\tau\, \mathrm{e}^{-\mathrm{i}[(\boldsymbol{k}-\boldsymbol{k}^\alpha)\cdot\boldsymbol{r}-\tilde{\omega}_n\tau]} G_0^\alpha(\boldsymbol{r}, \tau)\, \mathrm{e}^{Q^\alpha(\boldsymbol{r},\tau)} \tag{1}$$

where $\Theta^\alpha(\boldsymbol{k})$ is unity of $\boldsymbol{k} \in K^\alpha$ and vanishes otherwise, and

$$G_0^\alpha(\boldsymbol{r}, \tau) = \delta_\Lambda^{(d-1)}(\boldsymbol{r}_\perp) \left(\frac{-\mathrm{i}}{2\pi}\right) \frac{1}{r_\parallel + \mathrm{i}|\boldsymbol{v}^\alpha|\tau} \tag{2}$$

$$Q^\alpha(\boldsymbol{r}, \tau) = \frac{1}{\beta V} \sum_q f_q^{\mathrm{RPA}} \frac{1 - \cos(\boldsymbol{q}\cdot\boldsymbol{r} - \omega_m\tau)}{(\mathrm{i}\omega_m - \boldsymbol{v}^\alpha\cdot\boldsymbol{q})^2}. \tag{3}$$

Here β is the inverse temperature, V is the volume of the system, $q = [\boldsymbol{q}, \mathrm{i}\omega_m]$, and $r_\parallel = \boldsymbol{r}\cdot\hat{\boldsymbol{v}}^\alpha$, with $\hat{\boldsymbol{v}}^\alpha = \boldsymbol{v}^\alpha/|\boldsymbol{v}^\alpha|$. The fermionic Matsubara frequencies are denoted by $\tilde{\omega}_n = 2\pi[n + \frac{1}{2}]/\beta$, and the bosonic ones are $\omega_m = 2\pi m/\beta$. For length scales $|\boldsymbol{r}_\perp| \gg \Lambda^{-1}$ the function $\delta_\Lambda^{(d-1)}(\boldsymbol{r}_\perp)$ can be treated as a $(d-1)$-dimensional Dirac-δ function of the components of $\boldsymbol{r}$ orthogonal to $\hat{\boldsymbol{v}}^\alpha$. The effect of the interactions is contained in $Q^\alpha(\boldsymbol{r}, \tau)$, which depends exclusively on the random-phase approximation f_q^{RPA} for the interaction. In the functional-integral formulation of bosonization [6–9], $Q^\alpha(\boldsymbol{r}, \tau)$ can be interpreted simply as the usual *Debye–Waller factor* that arises in a Gaussian average. A result similar to equations (1)–(3) has also been obtained by Castellani *et al* [10] by means of a non-perturbative approach based on Ward identities [11].

We consider density–density interactions of the form $f_q = f_0(q_c/|\boldsymbol{q}|)^\eta\, \mathrm{e}^{-|\boldsymbol{q}|/q_c}$, with $\eta > 0$ and $q_c \ll k_F$. After lengthy but straightforward algebra we obtain from equation (3) for the leading asymptotic behaviour of the equal-time Debye–Waller factor at large distances

$$Q^\alpha(r_\parallel\hat{\boldsymbol{v}}^\alpha, 0) \sim \begin{cases} R_{d,\eta} & \text{for } \eta < 2(d-1) \\ -\gamma_d \ln(q_c|r_\parallel|) & \text{for } \eta = 2(d-1) \\ -\beta_{d,\eta}(q_c|r_\parallel|)^{\frac{\eta}{2}-d+1} & \text{for } \eta > 2(d-1) \end{cases} \tag{4}$$

where we have assumed for simplicity that the Fermi surface is spherically symmetric. Here $R_{d,\eta}$, γ_d and $\beta_{d,\eta}$ are finite real numbers that depend not only on d and η, but also in $F_0 \equiv \nu f_0$, where ν is the d-dimensional density of states at the Fermi energy. Explicit expressions for $R_{d,\eta}$, γ_d and $\beta_{d,\eta}$ can be written down in arbitrary d, but are omitted here for brevity, because the precise values for these quantities are not essential in this work. We would like to point out however, that in an arbitrary dimension there exists a critical value $\eta_d^* > 2(d-1)$ where β_{d,η_d^*} diverges. It is not clear if this divergence signals a physical instability of the metallic state, or simply arises from the inadequacy of the bosonization approach. For example, in $d = 1$ we obtain for $\eta > 0$

$$\begin{aligned}\beta_{1,\eta} &= \frac{\sqrt{F_0}}{2} \int_0^\infty \mathrm{d}x \frac{1 - \cos x}{x^{1+\eta/2}} \\ &= \frac{\sqrt{F_0}}{\eta} \Gamma\left(1 - \frac{\eta}{2}\right) \cos\left(\frac{\pi\eta}{4}\right).\end{aligned} \tag{5}$$

Because $\beta_{1,\eta} \to \infty$ for $\eta \to 4$, the static Debye–Waller factor is divergent for $\eta \geqslant 4$, so that $\eta_1^* = 4$. More generally, for singular interactions in arbitrary d it is easy to show by simple power counting that $\eta_d^* = 2(d+1)$. However, the finiteness of the static Debye–Waller factor does *not* imply that for $\tau \neq 0$ the function $Q^\alpha(r_\parallel\hat{\boldsymbol{v}}^\alpha, \tau)$ remains finite as well. For example, in $d = 1$ the standard bosonization procedure [12] leads to an integral over the term

$$\frac{\cos(q_\parallel r_\parallel)}{q_\parallel^{1+\eta/2}}\left[1-\exp\left(-\sqrt{F_0}q_c^{\eta/2}v_F|\tau|q_\parallel^{1-\eta/2}\right)\right] \sim \frac{\sqrt{F_0}q_c^{\eta/2}v_F|\tau|}{q_\parallel^{\eta}} \qquad (6)$$

where the expansion of the exponential for small $q_\parallel$ is justified as long as $\eta < 2$. Obviously, the singularity at small $q_\parallel$ is not integrable for $\eta \geqslant 1$, so that in $d = 1$ the full Green function can only be calculated via bosonization for $\eta < 1$. In the rest of this paper we shall assume that $0 < \eta < \eta_d^* = 2(d+1)$, so that the bosonization result for the equal-time Green function is free of divergences.

Given the equal-time Green function, we may calculate the momentum distribution $n_k = \frac{1}{\beta}\sum_n G(k, \mathrm{i}\tilde{\omega}_n)$ in the vicinity of the Fermi surface. Using equation (1) and shifting $k = k^\alpha + q$, it is easy to show that for $|q| \ll q_c$

$$\delta n_q^\alpha \equiv n_{k^\alpha - q} - n_{k^\alpha + q} = \frac{2}{\pi}\int_0^\infty \mathrm{d}r_\parallel \frac{\sin(q_\parallel r_\parallel)}{r_\parallel} \exp[Q^\alpha(r_\parallel \hat{v}^\alpha, 0)]. \qquad (7)$$

Note that δn_q^α depends only on the projection $q_\parallel$ of q that is normal to the Fermi surface, because this component corresponds to a crossing of the Fermi surface and can therefore be associated with a possible discontinuity. Because equation (4) is valid for $r_\parallel \gtrsim q_c^{-1}$, we separate from equation (7) the non-universal short-distance regime and obtain for $|q_\parallel| \ll q_c$ after rescaling the integration variables

$$\delta n_q^\alpha = z^\alpha \frac{q_\parallel}{q_c} + \frac{2}{\pi}\int_1^\infty \mathrm{d}x \frac{\sin((q_\parallel/q_c)x)}{x} \exp\left[Q^\alpha\left(\frac{x}{q_c}\hat{v}^\alpha, 0\right)\right] \qquad (8)$$

where the non-universal constant z^α is given by

$$z^\alpha = \frac{2}{\pi}\int_0^1 \mathrm{d}x \exp\left[Q^\alpha\left(\frac{x}{q_c}\hat{v}^\alpha, 0\right)\right]. \qquad (9)$$

For $|q_\parallel| \ll q_c$ we may substitute in the second term of equation (8) the asymptotic expansion of $Q^\alpha(\frac{x}{q_c}\hat{v}^\alpha, 0)$ for large x/q_c, see equation (4). For $\eta < 2(d-1)$ we obtain

$$\delta n_q^\alpha = \mathrm{e}^{R^\alpha}\mathrm{sign}(q_\parallel) \qquad \eta < 2(d-1). \qquad (10)$$

This is the usual Fermi liquid behaviour: the discontinuity of the momentum distribution at point k^α is given by the quasi-particle residue $Z^\alpha = \mathrm{e}^{R^\alpha}$. Because R^α is negative [7, 8], we have $0 < Z^\alpha < 1$. Note that for small $q_\parallel$ the first term in equation (8) is negligible. In the marginal case $\eta = 2(d-1)$ we obtain for $|q_\parallel| \ll q_c$

$$\delta n_q^\alpha \sim z^\alpha \frac{q_\parallel}{q_c} + \frac{2\,\mathrm{sign}(q_\parallel)}{\pi}\left|\frac{q_\parallel}{q_c}\right|^{\gamma_d} \int_{|q_\parallel|/q_c}^\infty \mathrm{d}x \frac{\sin x}{x^{1+\gamma_d}}. \qquad (11)$$

From this expression it is easy to show that to leading order

$$\delta n_q^\alpha \sim \begin{cases} \mathrm{sign}(q_\parallel)\left|\dfrac{q_\parallel}{q_c}\right|^{\gamma_d} & \text{for } \gamma_d \ll 1 \\ \dfrac{2}{\pi}\left(\dfrac{q_\parallel}{q_c}\right)\ln\left(\dfrac{q_c}{|q_\parallel|}\right) & \text{for } \gamma_d = 1 \\ \left(z^\alpha + \dfrac{2}{\pi(\gamma_d - 1)}\right)\dfrac{q_\parallel}{q_c} & \text{for } \gamma_d > 1. \end{cases} \qquad \eta = 2(d-1) \qquad (12)$$

An algebraic singularity in the momentum distribution is characteristic for Luttinger liquids [12]. Note, however, that for $\gamma_d = 1$ the singularity is only logarithmic, and that for $\gamma_d > 1$

the leading term is even analytic. Although, in this case, there are non-analyticities in the higher-order corrections, one may wonder whether for $\gamma_d > 1$ the bosonization approach is consistent. Recall that we have linearized the energy dispersion 'at the Fermi surface', thus implicitly assuming that the Fermi surface can somehow be defined. In the case of a Fermi liquid the finite discontinuity of the momentum distribution leads to a unique definition of the Fermi surface. For a Luttinger liquid one may define the Fermi surface as the set of points in momentum space where n_k has an algebraic singularity. However, for $\gamma_d > 1$ it is at least not quite satisfactory that one has to rely on asymptotically irrelevant corrections to define the Fermi surface. This point becomes even more questionable if we consider the case $\eta > 2(d-1)$. Then we obtain from equations (8) and (4)

$$\delta n_q^\alpha \sim z^\alpha \frac{q_\parallel}{q_c} + \frac{2}{\pi}\int_1^\infty \mathrm{d}x \frac{\sin\left(\frac{q_\parallel}{q_c}x\right)}{x} \exp[-\beta_{d,\eta} x^{\frac{\eta}{2}-d+1}]. \tag{13}$$

The crucial observation is now that *the stretched exponential vanishes faster than any power*, so that the integral can be done by expanding $\sin((q_\parallel/q_c)x)$ under the integral sign and exchanging the order of integration and summation. It immediately follows that δn_q^α is for $\eta > 2(d-1)$ *an analytic function of* $q_\parallel$. To leading order we have

$$\delta n_q^\alpha = \left[z^\alpha + \frac{2}{\pi}\int_1^\infty \mathrm{d}x \exp[-\beta_{d,\eta} x^{\frac{\eta}{2}-d+1}]\right] \frac{q_\parallel}{q_c} + \mathrm{O}(q_\parallel^2). \tag{14}$$

If $\beta_{d,\eta}$ or $\frac{\eta}{2} - d + 1$ is small, the second term is dominant, because then the integral is determined by the large-x regime. Then we may extend the lower limit to zero and obtain

$$\delta n_q^\alpha = \frac{4}{\pi} \frac{\Gamma\left(\frac{2}{\eta - 2(d-1)}\right) \beta_{d,\eta}^{-2/(\eta-2(d-1))}}{\eta - 2(d-1)} \frac{q_\parallel}{q_c} + \mathrm{O}(q_\parallel^2) \qquad \eta > 2(d-1). \tag{15}$$

Hence, there is no singularity whatsoever in the momentum distribution, so that a sharp Fermi surface simply cannot be defined. The complete destruction of the Fermi surface in strongly correlated Fermi systems is certainly not a special feature of the singular interactions studied in the present work. For example, models with correlated hopping [13, 14] show similar behaviour.

The obvious question now is whether for $\eta > 2(d-1)$ the bosonization approach is consistent or not. Before addressing this question, let us briefly recall that in disordered systems the situation is precisely the same [15]. Here also the average momentum distribution in the vicinity of the Fermi surface of the clean system does not have any singularities. In this case the thickness of the shell where n_k drops from unity to zero is given by the inverse mean free patch ℓ^{-1}. Because in good metals $\ell^{-1} \ll k_F$, the 'Fermi surface' is defined in the sense that outside the limits of a thin shell in momentum space the derivative of n_k is negligibly small. Because in the laboratory impurities can never be completely eliminated, this definition of the Fermi surface does justice to the experimental reality, although it is impossible to define a surface in the strict mathematical sense. Clearly, the 'Fermi surface' in the present problem should be defined analogously: as long as the thickness k_S of the shell where the momentum distribution varies appreciably is small compared with k_F, it is meaningful to talk about a *smeared Fermi surface*, or, more accurately, a *Fermi shell*. The condition $k_S \ll k_F$ is sufficient to make the bosonization approach internally consistent, because then it does not matter at which location within the Fermi shell the non-interacting energy dispersion has been linearized. This point of view has also been emphasized in the classic paper by Tomonaga [12].

The properties of strongly correlated quantum liquids are not only of academic interest. Physical manifestations of such an unusual metallic state might have been observed in the normal-state of the cuprate superconductors [16], or in half-filled quantum Hall systems [17]. Recent theoretical models for these systems involve the coupling between electrons and transverse gauge fields, which at the fermionic level leads to an effective current–current interaction. In the perturbative calculation of the fermionic self-energy this rather singular interaction gives rise to divergences which are stronger than logarithmic. The singular nature of the current–current interaction is essentially a consequence of gauge invariance, which implies that, in the absence of superconducting instabilities, the gauge field cannot be screened in the static limit [18, 19]. Recently this problem and its generalizations to arbitrary dimension d has been re-examined by a number of authors [20–28]. Although the applicability of the higher dimensional bosonization approach to this problem has been questioned [24, 25], there exist other independent non-perturbative calculations which confirm the bosonization result [20]. It is perhaps fair to say that at present the issue is far from being settled. For electrons coupled to the Maxwell field the higher-dimensional bosonization approach [26, 28] implies that $d = 3$ is a marginal dimension in the problem, in agreement with the renormalization group calculations [21, 27]. For $d < 3$ bosonization predicts $Q^{\alpha}(r_{\parallel}\hat{v}^{\alpha}, 0) \propto -(\kappa_d r_{\parallel})^{(3-d)/3}$, where the momentum scale κ_d is given in [28]. From our analysis given above it is clear that the stretched exponential behaviour implies that below three dimensions the coupling to the transverse gauge field completely washes out the Fermi surface.

In summary, we have shown that in strongly correlated quantum liquids the momentum distribution is an analytic function close to the non-interacting Fermi surface. In these systems the concept of a Fermi surface must be replaced by a Fermi shell. The, perhaps, most important physical realization of such a system are half-filled quantum Hall systems. Although recent experiments suggest that in these systems there is a well-defined Fermi surface [29], the finite smearing scale k_S might be beyond experimental resolution. We have also argued that, at least for not too singular interactions, the bosonization approach remains consistent as long as the thickness k_S of the Fermi shell is small compared with k_F.

I would like to thank Cornelia Buhrke, Guillermo Castilla, and Jean Lapointe for enjoying with me the 1995 APS March Meeting in San Jose, where this work was conceived. I am also grateful to K Schönhammer for many discussions and fruitful collaborations.

References

[1] Bares P-A and Wen X-G 1993 *Phys. Rev.* B **48** 8636
[2] Luther A 1979 *Phys. Rev.* B **19** 320
[3] Haldane F D M 1992 *Helv. Phys. Acta.* **65** 152; 1994 Perspectives in many-particle physics *Proc. Int. School of Physics 'Enrico Fermi', Course 121* (Amsterdam: North Holland)
[4] Houghton A and Marston J B 1993 *Phys. Rev.* B **48** 7790
Houghton A, Kwon H-J and Marston J B 1994 *Phys. Rev.* B **50** 1351
Houghton A, Kwon H-J, Marston J B and Shankar R 1994 *J. Phys. C: Solid State Phys.* **6** 4909
Kwon H-J, Houghton A and Marston J B1995 cond-mat/9501067
[5] Castro Neto A H and Fradkin E 1994 *Phys. Rev. Lett.* **72** 1393; 1994 *Phys. Rev.* B **49** 10 877; 1995 *Phys. Rev.* B **51** 4084
[6] Fröhlich J, Götschmann R and Marchetti P A 1995 *J. Phys. A: Math. Gen.* **28** 1169
[7] Kopietz P and Schönhammer K 1995 *Z. Phys.* B cond-mat/9502088
Kopietz P and Hermisson J and Schönhammer K 1995 *Phys. Rev.* B cond-mat/9502089
[8] Kopietz P 1995 *Proc. Raymond L Orbach Symp.* ed D Hone (Singapore: World Scientific) to appear; 1995 *Springer Tracts in Modern Physics* Habilitationsschrift, Universität Göttingen, cond-mat/9506132, in preparation
[9] Fogedby H C 1976 *J. Phys. C: Solid State Phys.* C **9** 3757
Lee D K K and Chen Y 1988 *J. Phys. A: Math. Gen.* A **21** 4155

[10] Castellani C, Di Castro C and Metzner W 1994 *Phys. Rev. Lett.* **72** 316

[11] Metzner W and Di Castro C 1993 *Phys. Rev.* B **47** 16107

[12] Tomonaga S 1950 *Prog. Theor. Phys.* **5** 544
Luttinger J M 1963 *J. Math. Phys.* **4** 1154
Mattis D C and Lieb E H 1965 *J. Math. Phys.* **6** 304

[13] Hirsch J E 1989 *Physica* **158C** 326

[14] Bariev R Z, Klümper A, Schadschneider A and Zittartz J 1993 *J. Phys. A: Math. Gen.* **26** 1249, 4863

[15] Abrikosov A A, Gorkov L P and Dzyaloshinski I E 1963 *Methods of Quantum Field Theory in Statistical Physics* (New York: Dover)

[16] Ioffe L B and Larkin A I 1989 *Phys. Rev.* B **39** 8988
Lee P A and Nagaosa N 1992 *Phys. Rev.* B **46** 5621
Blok B and Monien H 1993 *Phys. Rev.* B B **47** 7312

[17] Halperin B I, Lee P A and Read N 1993 *Phys. Rev.* B **47** 7312

[18] Holstein T, Norton R E and Pincus P 1973 *Phys. Rev.* B **8** 2649

[19] Pethick C J, Baym G and Monien H 1989 *Nucl. Phys.* A **498** 313

[20] Khveshchenko D V and Stamp P C E 1993 *Phys. Rev. Lett.* **71** 2118

[21] Gan J and Wong E 1993 *Phys. Rev. Lett.* **71** 4226

[22] Plochinski J 1994 *Nucl. Phys.* B **422** 617

[23] Nayak C and Wilczek F 1994 *Nucl. Phys.* B **417** 359

[24] Ioffe L B, Lidsky D and Altshuler B L 1994 *Phys. Rev. Lett.* **73** 472
Altshuler B L, Ioffe L B and Millis A J 1994 *Phys. Rev.* B **50** 14048

[25] Castellani C and Di Castro C 1994 *Physica* **235–240C** 99

[26] Kwon H-J, Houghton A and Marston J B 1994 *Phys. Rev. Lett.* **73** 284

[27] Chakravarty S, Norton R E and Syljasen O F 1995 *Phys. Rev. Lett.* **74** 1423

[28] Kopietz P 1994 *Preprint* cond-mat/ 9503090

[29] Willet R L *et al* 1993 *Phys. Rev. Lett.* **71** 3846
Willet R L *et al* 1993 *Phys. Rev.* B **47** 7344
Kang W *et al* 1993 *Phys. Rev. Lett.* **71** 3850
Goldman V J, Su B and Jain J K 1994 *Phys. Rev. Lett.* **72** 2065